Computer Science & Maths

The Interdisciplinary Importance of Computer Science

PROF. CH. POSTHOFF

Department of Mathematics & Computer Science, UWI, Trinidad & Tobago, cposthoff@fans.uwi.tt

Introduction

Historically, Computer Science has been strongly associated with Mathematics. During the 60's and 70's of the last century, Computer Science was nurtured at Universities in Departments of Mathematics. By the late 70's and early 80's, however, Computer Science had matured sufficiently and many universities spun out Computer Science as a discipline distinct from Mathematics. Its position in the organization structure was not uniform and several models emerged: a Department of Computer Science within the Science Faculty or within the Faculty of Engineering or even within Social Sciences (Dept. of Information Systems). By the 90's, the model that emerged as the preferred placement was a School of Computer Sciences (names vary) with links to Science, Engineering, Social Studies, and even Medical Sciences (Bioinformatics and Medical Computing).

These developments are especially crucial because the level of one discipline is strongly dependent on another discipline, and only the efficient cooperation and combination of Computer Science and other disciplines results in a modern education and application of this knowledge.

The Interdisciplinary Importance

The guiding principle will be that ***the level of university education is only as good as the level of Computer Science application and knowledge***. The paper will give some examples to underline this principle and show the responsibility of academic staff to cope with this responsibility. It can be seen very clearly that the production of transparencies, course material etc. is only a first and tiny step. And it must also be understood that this is only a small set of examples, an exhaustive survey would require much more time and space.

1- Mathematics

At present very powerful systems for Mathematics are available, such as MAPLE, MATHEMATICA, MATLAB, MATHCAD, among others. It still can be discussed what really has to be done with these systems in teaching Mathematics, however, there is no doubt at all that courses must be available for other Science disciplines (such as Physics, Biology, Chemistry,…) that present and use these systems. Based on these systems, **Mathematics is more applicable than ever before**. However, no longer the computational part is the core of applications, it is most important to express a problem in Physics, … in a mathematical format - hence, ***mathematical modeling*** is crucial, and thereafter, it is very necessary to understand the solutions and to use them properly. Thus a Science Faculty has an urgent need of courses in **Modeling and Simulation** or **Scientific Computing**, compulsory for all science students. This approach can immediately be extended to all the Engineering disciplines that are also very related to Mathematics. Special systems are dealing with **Statistics** and **Financial Mathematics**, they must also be used, however, in a narrower environment.

2- Other Science Disciplines

In addition to the influence of Mathematics to the respective Science disciplines or by using Mathematics as a bridge, we see the development of special disciplines that represent exactly the combination of the Science discipline and/or Mathematics with Computer Science. The development of areas such as *Computational Physics, Computational Chemistry, Bioinformatics, DNA-Computing, Quantum Computing* can be seen, at present the teaching of these sub-disciplines or even an introduction do not exist.

3- Medicine

The role of Computer Science in *Medicine* is rapidly growing, many areas of *Medicine* get more and more computerized. Very often these developments are based on the union of Medicine, different developments in Electronics and computer-based methods. Internationally developments such as ***Telemedicine*** or ***Medical Informatics*** can be found. Very interesting and useful in the very near future is, for instance, the cooperation between ***Artificial Intelligence*** or ***Soft Computing*** methods and Medicine. Learning from examples (that exist in Medicine in large numbers) and the creation of very powerful ***Diagnostic Systems*** are only one example. All kinds of methods for ***image processing*** are another rapidly growing field. The arrival and use of digital cameras plays an important role even in the daily life. There is no doubt that students of Medicine must be educated in these areas.

4- Education

An urgent need to educate Computer Science teachers as well as to train all teachers in the use of computers for educational purposes. The development of Computer Science Didactics and the ***Didactics of computer-based instruction and teaching*** need much more attention. These areas are mainly not existing or restricted to the use of some commercial systems. ***Distance Education*** is another important factor that can considerably extend University education to distant places and islands and improve considerably the quality of education and of life in distant areas.

5- Social Sciences

The required overlap with Computer Science (*Information systems, E-Commerce, Database Systems Applications, Internet Technologies, Financial Systems, Security Issues)* is only rudimentarily discussed and defined. Internationally, however, we can see enormous developments in many of these areas. ***E-Government*** is another "hot topic" that requires efforts to be implemented and that can considerably improve the quality of the Public Service and many branches of the business life. The use of Computer Science in the legal system is a very broad part of these issues and can considerably improve the efficiency of this system.

6- Scientific Writing

This is another very typical situation in this changing world of education based on computerization. Any publisher requires nowadays manuscripts that are more or less ready for publication. Science-related publications rely heavily on Mathematics, however, typing of Mathematics requires special knowledge and understanding. All our graduate students should acquire this knowledge compulsorily, based on advanced type-setting systems to make them competitive in this area.

7- Conclusions

These are only some examples that show the enormous responsibility of each staff member to acquire the necessary knowledge and to work, more than ever before, in an interdisciplinary way for the improvement of the quality of the level of our university education.

Parallel Mining Of Association Rules

THOMAS WESSEL

University of Technology, 237 Old Hope Road, Kingston 6, Jamaica - thomaswe@scis.nova.edu

Abstract

In this paper we present a parallel algorithm for the mining of association rules. We implemented a parallel algorithm that used a lattice approach for mining association rules. The Dynamic Distributed Rule Mining (DDRM) is a lattice-based algorithm that partitions the lattice into sub-lattices to be assigned to processors for processing and identification of frequent item sets. We implemented the DDRM using a dynamic load balancing approach to assign classes to processors for analysis of these classes in order to determine if there are any rules present in them. Experimental results show that DDRM utilizes the processors efficiently and performed better than the prefix-based algorithm that uses a static approach to assign classes to the processors. The DDRM algorithm scales well and shows good speedup.

Introduction

Many organizations are now finding it feasible economically to create ultra large databases of business and scientific data. This is made possible by the availability of inexpensive storage devices and developments in data capture technology (Agrawal & Shafer, 1996). Bar-code technology has made it possible to collect and store large amounts of sales data in retail organizations. The records associated with retail data are typically made up of transaction data and items bought in the transaction. These databases are viewed by organizations as important pieces of marketing infrastructure. Organizations are now using this data for the mining of association rules.

Problem Statement

The goal of data mining is the discovery of unknown patterns in large databases using efficient techniques to find these rules.

An association rule is of the form, “87% of customers that purchase a computer also purchase a printer.” Agrawal, Imielinski and Swami (1993) first introduced the problem of mining association rules. According to Zaki (2000) the search space for the discovery of all frequent associations in very large databases is exponential in the number of database attributes. In addition this is further complicated by I/O requirements for the millions of database objects.

In this paper we present the Dynamic Distributed Rule Mining (DDRM) algorithm that uses a lattice to represent the search space for the generation of the frequent item sets. DDRM partitions the search space and assigns each partition dynamically to the next available processor. An evaluation of the algorithm was carried out and its performance relative to the prefix-based with bottom-up search, which is a parallel algorithm for mining of association rules, was also determined (Zaki, 2000).

The DDRM algorithm does not require any special architecture for its implementation. It is designed to operate on an existing LAN where the PCs can be added to the cluster and used to participate in the computations of the classes. The database of transactions can also be distributed over the network. This flexibility of the algorithm will result in significant savings to the organization as it uses the resources that are already available within the organization. This reduction in cost is due to the fact that there is no need for specialized architecture and makes the algorithm attractive to an organization that currently operates a network with databases distributed over it.

The Message Passing Interface (MPI) model consists of P processors each with local memory, connected over a communication network. MPI facilitates communications among a set of processors that have only local memory through the mode of sending and receiving of messages.

Experimental Results

The DDRM algorithm was developed and implemented using C/C++ as the programming language on an Ethernet LAN consisting of 7 workstations and one server. Each workstation on the network is an AMD Athlon XP 2800+ with 512 Mbytes of memory. The processors are interconnected via a 10/100 Mbps switch. The switch used 100BASE-T (Fast Ethernet) technology, which provided greater bandwidth and improved the client/server response time. For communications we used the message passing interface (MPI). We used the windows message passing interface (WMPI) for 8 workstations from Critical Software Ltd to implement our algorithm

We used the 1987 census data from the Statistical Institute of Jamaica to generate the data used in this experiment. The size of the database was 25 Mbytes with 1.1 million records. The support count used was 8% with a confidence of 50%. The experiment was conducted by partitioning the database among the processors.

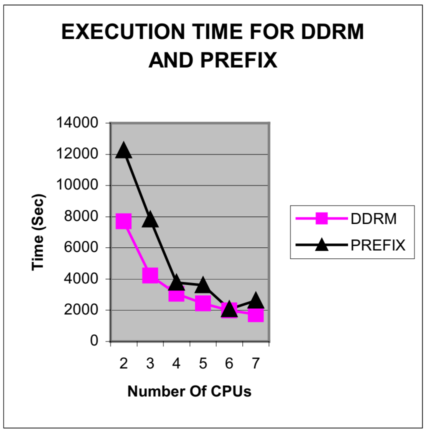


***Figure 1** Execution Time for DDRM and Prefix*

In Figure 1 we show a plot of the response time on the vertical axis and the number of processors on the horizontal axis. The database was partitioned based on the number of processors. The search space was partitioned into 32 classes, which were assigned dynamically to the processors participating in the cluster. It can be seen that as we increase the number of processors the response time decreases as well. We compare the response time for DDRM with that of the Prefix-based algorithm. The response time for DDRM is better than that for the Prefix-based.

We also obtained a maximum speedup of approximately 4.5. The DDRM algorithm also scales well.

Conclusion

This paper has presented the Dynamic Distributed Rule Mining (DDRM), which is a lattice-based algorithm that partitions the lattice into sublattices to be assigned to processors for processing and identification of frequent itemsets. We implemented the DDRM using a dynamic load balancing approach to assign classes to processors for analysis of these classes in order to determine if there are any rules present in them. Experimental results show that DDRM utilizes the processors efficiently and performed better than the prefix-based algorithm that uses a static approach to assign classes to the processors. The DDRM algorithm scales well and shows good speedup.

References

Agrawal, R., Imielinski, T., & Swami, A. (1993). Mining association rules between sets of items in large databases. *Proceedings of ACM SIGMOD International Conference on Management of Data*, 207-216.

Agrawal, R., & Shafer, J. C. (1996). Parallel mining of association rules. *IEEE Transaction on Knowledge and Data Engineering,* 8 (6), 962-969.

Zaki, M. J. (2000). Scalable algorithms for association mining. *IEEE Transactions on Knowledge and Data Engineering*, 12(3), 372-390.

Boundary controllability with constraints on the state

Gisèle Massengo Mophou and Ousseynou Nakoulima
Laboratoire de Mathématiques et Informatique
Université des Antilles et de La Guyane, Campus Fouillole
97159 Pointe-à-Pitre Guadeloupe (FWI)
Gisele.Mophou@univ-ag.fr, Ousseynou.Nakoulima@univ-ag.fr

Abstract

We consider a boundary controllability problem with a finite number of constraints on the state. Interpreting each constraint, we transform the problem into an equivalent controllability problem with constraint on the control. Then using an inequality of observability which derives from the boundary inequality of Carleman, we prove that the equivalent controllability problem has a solution.

Key-words : Heat equation, controllability, inequality of Carleman.

1 Introduction

Let N, $M \in \mathbf{N}^*$ and Ω be a bounded open subset of $\mathbb{R}^N$ with boundary Γ of class $\mathcal{C}^2$. Let $\omega \subset \Gamma$ be an open non empty subset. For a time $T > 0$, we set $Q = \Omega \times (0,T)$ and $\Sigma = \Gamma \times (0,T)$ and we consider the semilinear heat equation:

$$\left\{ \begin{array}{rcll} \dfrac{\partial y}{\partial t} - \Delta y + a_0 y & = & 0 & \text{in} \quad Q, \\ y & = & v\chi_\omega & \text{on} \quad \Sigma, \\ y(0) & = & y^0 & \text{in} \quad \Omega \end{array} \right. \tag{1}$$

where $a_0 \in L^\infty(Q)$, $y^0 \in L^2(\Omega)$, the control v belongs to $L^2(\Gamma \times (0,T))$ and χ_ω represents the characteristic function of the control set ω.
Since $v\chi_\omega \in L^2(\Gamma \times (0,T))$, $a_0 \in L^\infty(Q)$ and $y^0 \in L^2(\Omega)$, one can prove using transposition method (see [5]) that the problem (1) has a unique solution $y \in L^2(Q)$.
In this paper, we are interested in the following boundary controllability problem with a finite number of constraints: *Given $e_i \in L^2(Q)$, $1 \le i \le M$ and $y^0 \in L^2(\Omega)$, find a control $v \in L^2(\Gamma \times (0,T))$ such that the solution of (1) satisfies*

$$\int_0^T \int_\Omega e_i \, dx \, dt = 0, \quad 1 \le i \le M. \tag{2}$$

There is a large literature on controllability of the heat equations. Let us mention briefly some of the existing works.
In the context of linear heat equation, G. Lebeau and L. Robbiano in [4], solved the null controllability using Fourier series and sharp estimates on the eigenfunctions of the Laplacian obtained by means of Carleman inequalities. In the nonlinear case, A. Fursikov A. and O. Yu. Imanuvilov in [2], showed using a Carleman's estimate that, when the control acts on the boundary, null controllability holds for bounded continuous and sufficiently small initial data.
There is a large literature on approximate controllability problem as well. For instance in [1], C. Fabre, J.-P. Puel and E. Zuazua proved, using the minimization of a particular functional, that the following approximate controllability holds: *Given y^0, $y^1 \in L^2(\Omega)$ and $\alpha > 0$, find a control $v \in L^2(Q)$ such that the solution of (1) satisfies* $\|y(T) - y^1\|_{L^2(\Omega)} \le \alpha$. The method is supported by a unique continuation theorem.
Recently, O. Nakoulima proved [6], using an adapted Carleman inequality, that the following null controllability with constraint on the control holds: *Given an open non empty subset γ of Ω, a finite dimensional subspace Y of $L^2(\gamma \times (0,T))$ and $y^0 \in L^2(\Omega)$, find a control $v \in Y^\perp$, the orthogonal of Y in $L^2(\gamma \times (0,T))$, such that the solution of (1) satisfies $y(T) = 0$ in Ω.*

In this paper, we prove by means of Carleman inequality that the controllability problem with constraints on the state (1) and (2) holds.
Without loss of generality, we can assume that

$$\left\{ \begin{array}{l} \{e_1\chi_\omega, \cdots, e_M\chi_\omega\} \text{ is a family of} \\ \quad M \text{ independent functions.} \end{array} \right. \tag{3}$$

The main result of the paper is the following theorem.

Theorem 1.1 *Assume that the hypotheses above on Ω and ω are satisfied. Then for every $e_i \in L^2(Q)$, $1 \le i \le M$, every $T > 0$, there exists $v \in L^2(\Gamma \times (0,T))$ such that the solution of (1) sastifies (2). Moreover, there exists*

$C = C(\Omega, \omega, \|a_0\|_{L^\infty(Q)}, \sqrt{M}) > 0$ *such that*

$$\|v\|_{L^2(\Sigma)} \le C\|y^0\|_{L^2(\Omega)}. \qquad (4)$$

The rest of the paper is devoted to the proof of Theorem 1.1.

2 Proof of Theorem 1.1

To prove Theorem 1.1, we proceed in three steps.
Step 1. We prove that controllability with constraints on the state (1) and (2) is equivalent to controllability with constraint on the control.
To this end, we interpret the relations (2), using the notion of adjoint. More precisely, for each e_i, $1 \le i \le M$, we consider the ajoint system:

$$\left\{ \begin{array}{rcll} -\dfrac{\partial p_i}{\partial t} - \Delta p_i + a_0 p_i & = & e_i & \text{in } Q, \\ p_i & = & 0 & \text{on } \Sigma, \\ p_i(T) & = & 0 & \text{in } \Omega. \end{array} \right. \qquad (5)$$

Since $a_0 \in L^\infty(Q)$ and $e_i \in L^2(Q)$, the problem (5) admits a unique solution p_i in
$H^{2,1}(Q) = L^2(0,T;H^2(\Omega)) \cap H^1(0,T;L^2(\Omega))$.

Lemma 2.1 *Assume (3). Then the functions* $\dfrac{\partial p_i}{\partial \nu}\chi_\omega$, $1 \le i \le M$ *are linearly independent.*

Proof. We use the unique continuation theorem due to Mizohata [7] and the fact that the functions $e_i\chi_\omega$, $1 \le i \le M$ are linearly independent. ■

From now on, we assume that the functions

$$\{\frac{\partial p_i}{\partial \nu}\chi_\omega, 1 \le i \le M\} \text{ are orthonormal.} \qquad (6)$$

Remark 1 *If the functions* $\dfrac{\partial p_i}{\partial \nu}\chi_\omega$, $1 \le i \le M$ *are not orthonormal, it suffices to apply the algorithm of Gram-Schmidt on the* $\dfrac{\partial p_i}{\partial \nu}\chi_\omega$ *to obtain a family of orthonormal functions since the functions* $\dfrac{\partial p_i}{\partial \nu}\chi_\omega$, $1 \le i \le M$ *are linearly independent.*

Thus, multiplying both sides of the differential equation in (1) by p_i and integrating by parts in Q, we have using (5),

$$\int_0^T \int_\omega v \frac{\partial p_i}{\partial \nu} d\Gamma\, dt = -\int_0^T \int_\Omega y e_i\, dx\, dt + \int_\Omega y^0 p_i(0)\, dx, \quad 1 \le i \le M.$$

Therefore, taking into account the conditions (2), we obtain

$$\int_0^T \int_\omega v \frac{\partial p_i}{\partial \nu} d\Gamma\, dt = \int_\Omega y^0\, p_i(0)\, dx,\ 1 \le i \le M. \quad (7)$$

Let

$$\mathcal{U} = \text{Span}\left(\frac{\partial p_1}{\partial \nu}\chi_\omega, \cdots, \frac{\partial p_M}{\partial \nu}\chi_\omega\right), \qquad (8)$$

be the real vector subspace of $L^2(\Gamma \times (0,T))$ generated by the M independent functions $\dfrac{\partial p_i}{\partial \nu}\chi_\omega$.
Then, there exists a unique $u_0 \in \mathcal{U}$ such that

$$\int_\Omega y^0 p_i(0) dx = \int_0^T \int_\omega u_0 \frac{\partial p_i}{\partial \nu}\, dx dt,\ 1 \le i \le M. \quad (9)$$

Consequently, (7) holds if and only if:

$$v - u_0\chi_\omega = u\chi_\omega \in \mathcal{U}^\perp \qquad (10)$$

where $\mathcal{U}^\perp$ is the orthogonal of $\mathcal{U}$ in $L^2(\omega \times (0,T))$.
Thus,

$$v = (u_0 + u)\chi_\omega \qquad (11)$$

and (1) becomes

$$\left\{ \begin{array}{rcll} \dfrac{\partial y}{\partial t} - \Delta y + a(z) y & = & 0 & \text{in } Q, \\ y & = & (u_0+u)\chi_\omega & \text{on } \Sigma, \\ y(0) & = & y^0 & \text{in } \Omega. \end{array} \right. \qquad (12)$$

Conversely, assume that $a_0 \in L^\infty(Q)$, $y^0 \in L^2(\Omega)$ and $e_i \in L^2(Q)$, $1 \le i \le M$ are given. Then solving (5), we get the functions p_i, $1 \le i \le M$. Next, we define $\mathcal{U}$ as in (8) and we denote by $\mathcal{U}^\perp$ its orthogonal in $L^2(\omega \times (0,T))$. Multiplying both sides of the differential equation in (12) by p_i and integrating by parts in Q, we have

$$\int_0^T \int_\omega (u_0 + u) \frac{\partial p_i}{\partial \nu}\, d\Gamma\, dt = -\int_0^T \int_\Omega y\, e_i\, dx\, dt + \int_\Omega y^0\, p_i(0)\, dx, \quad 1 \le i \le M.$$

Then, since $(u_0(z), u(z)) \in \mathcal{U} \times \mathcal{U}^\perp$ is such that (9) and (10) hold, this latter identity is reduced to (2).
In short, we have proved in the first step that the controllability problem with constraints on the state: *Given* $a_0 \in L^\infty(Q)$, $y^0 \in L^2(\Omega)$ *and* $e_i \in L^2(Q)$, $1 \le i \le M$, *find* v *in* $L^2(\Gamma \times (0,T))$ *such that the solution* y *of (1) sastifies (2)* is equivalent to the controllability problem with constraint on the control: *Given* $a_0 \in L^\infty(Q)$, $u_0 \in \mathcal{U} \subset L^2(\omega \times (0,T))$ *and* $y^0 \in L^2(\Omega)$, *find* $u \in L^2(\omega \times (0,T))$ *so that*

$$u \in \mathcal{U}^\perp \qquad (13)$$

and such that $y = y(u)$ *is solution of (12).*

Step 2. According to (10), the controllability problem with constraint on the control (13) and (12) has an infinitely many solutions. Since among the controls $u \in \mathcal{U}^{\perp}$ such that $y = y(u)$ is solution of (12), $u = 0$ is the control which has the minimal norm in $L^2(\omega \times (0,T))$, we choose

$$u = 0. \tag{14}$$

Therefore from (11), we have

$$v = u_0 \chi_\omega. \tag{15}$$

Thus, we can find $v \in L^2(\Gamma \times (0,T))$ verifying (15) such that $y = y(v)$ which is solution of (1) satisfies (2).

Step 3. We show that the control v given by (15) verifies (4).

First of all, let us recall the following inequality of observability which derives from the boundary Carleman inequality (see [3]):

Proposition 2.1 *There exists* $C = C(\Omega, \gamma, \|a_0\|_{L^\infty(Q)}) > 0$ *such that, for any* ρ *verifying* $\rho \in L^2(Q)$, $-\dfrac{\partial \rho}{\partial t} - \Delta\rho + a_0\rho \in L^2(Q)$ *and* $\rho = 0$ *on* Σ,

$$\int_\Omega |\rho(0)|^2\,dx \le C\left[\int_0^T\int_\gamma \left|\frac{\partial\rho}{\partial\nu}\right|^2 d\Gamma\,dt + \int_0^T\int_\Omega \left|-\frac{\partial\rho}{\partial t} - \Delta\rho + a_0\rho\right|^2 dx\,dt\right]. \tag{16}$$

Now, since u_0 verifies (9), applying the inequality of Cauchy-Schwartz to right hand side of this identity, we obtain

$$\left|\int_0^T\int_\omega u_0 \frac{\partial p_i}{\partial\nu} d\Gamma\,dt\right| \le \|y^0\|_{L^2(\Omega)}\|p_i(0)\|_{L^2(\Omega)}.$$

Therefore, a_0 being in $L^\infty(Q)$ and $p_i = 0$ on Σ, using (5) and the inequality of observability (16), we have

$$\left|\int_0^T\int_\omega u_0 \frac{\partial p_i}{\partial\nu} d\Gamma\,dt\right| \le C\|y^0\|_{L^2(\Omega)}\left(\left\|\frac{\partial p_i}{\partial\nu}\right\|^2_{L^2(\omega\times(0,T))} + \|e_i\|^2_{L^2(Q)}\right)^{1/2}$$

where $C = C(\Omega, \omega, \|a_0\|_{L^\infty(Q)})$. Hence, we deduce, since $\left\|\dfrac{\partial p_i}{\partial\nu}\right\|_{L^2(\Sigma)} = 1$,

$$\left|\int_0^T\int_\omega u_0 \frac{\partial p_i}{\partial\nu} d\Gamma\,dt\right| \le C\sqrt{1 + \|e_i\|^2_{L^2(Q)}}\|y^0\|_{L^2(\Omega)}.$$

Consequently,

$$\|u_0\|^2_{L^2(\omega\times(0,T))} = \sum_{i=1}^M \left|\int_0^T\int_\omega u_0\frac{\partial p_i}{\partial\nu}d\Gamma\,dt\right|^2 \le C^2\left(M + \sum_{i=1}^M \|e_i\|^2_{L^2(Q)}\right)\|y^0\|^2_{L^2(\Omega)}.$$

Therefore, v verifying (15), we deduce from this latter inequality the estimate (4). ■

References

[1] Fabre C., Puel J.-P., Zuazua E., Approximate controllability of the semilinear heat equation, Proc. Royal Soc. Edinburgh, 125A, (1995); pp. 31-61.

[2] Fursikov A. and Imanuvilov O. Yu., Controllability of evolution equations. Lecture Notes, Research Institute of Mathematics, Seoul National University, Korea, (1996).

[3] Imanuvilov O. Yu., Controllability of parabolic equations. Sbornik Mathematics 186 : 6 (1995); pp. 879-900.

[4] Lebeau G. and Robbiano L., Contrôle exacte de l'équation de la chaleur, Comm. P.D.E., 20, (1995); pp. 335-356.

[5] Lions J.-L. Optimal control of systems governed by partial differential equations. Springer, NY, (1971).

[6] Nakoulima O., Optimal control for distributed systems subject to null-controllability. Application to discriminating sentinels. ESAIM: Control, Optimisation and Calculus of Variations, Vol.13, 4(2007); pp. 623-638.

[7] Mizohata S., Unicité du prolongement des solutions pour quelques équations différentielles paraboliques, Memoirs of sciences University of Kyoto, 31(1958); pp. 219-239.

Characteristic Cauchy problem for nonlinear unidirectional wave equation

J.-A. MARTI

Laboratoire GTSI – Université des Antilles et de la Guyane, Guadeloupe - jamarti@univ-ag.fr

During the last three decades, theories of nonlinear generalized functions have been developed by many authors. These theories have proved their efficiency to pose and solve differential problems with irregular data and operators or characteristic problems.

In this paper we consider this last question, not from a general point of view, but with a thorough study of some typical example which has no solution in the classical theory.

The non linear characteristic initial value (or Cauchy) problem we choose to study can be written

$$\begin{cases} \dfrac{\partial u}{\partial t} + (\varphi' \otimes 1_x)\dfrac{\partial u}{\partial x} = F(.,.,u) \\ u\,|_S = f \end{cases} \tag{CIVP}$$

where $S = \Gamma_\varphi$ is the characteristic curve given by $x = \varphi(t)$, φ a smooth function, with $\varphi(0) = 0$ and $\varphi' > 0$ on $\mathbb{R}$ and f a function in $\mathrm{C}^\infty(\mathbb{R})$. We suppose, in addition, that F is a continuous function of all its arguments verifying a Lipschitz condition given by the following hypothesis

$$\forall K \Subset \mathbb{R}^2, \sup_{(t,x)\in K} |\partial_z F(t,x,z)| = m_K < +\infty$$

where the symbol $K \Subset \mathbb{R}^2$ means that K is a compact subset of $\mathbb{R}^2$. We know that the problem is ill posed, and cannot be solved by any classical procedure.

We approach (CIVP) by the one parameter family

$$\begin{cases} \dfrac{\partial u_\varepsilon}{\partial t} + (\varphi' \otimes 1_x)\dfrac{\partial u_\varepsilon}{\partial x} = F(.,.,u_\varepsilon) \\ u_\varepsilon(t,\varphi(t)+\varepsilon t) = f(t) \end{cases} \tag{CIVP$_\varepsilon$}$$

and we consider the family $(u_\varepsilon)_\varepsilon$ of the solutions to (CIVP$_\varepsilon$). They have to verify the integral

$$u_\varepsilon(t,x) = f[x-\varphi(t)] + \int_0^t F[\tau, x, u_\varepsilon(\tau,x)]\,d\tau. \tag{1}$$

Picard's procedure to solve (1) is to set up a sequence of successive approximation $u_{\varepsilon,n}$ defined by the formula

$$u_{\varepsilon,n+1}(t,x) = f[x-\varphi(t)] + \int_0^t F[\tau, x, u_{\varepsilon,n}(\tau,x)]\,d\tau.$$

We prove the uniform convergence on any compact set of the sequence $u_{\varepsilon,n}$ to the solution u_ε of (CIVP$_\varepsilon$). Then, we can construct the generalized function belonging to the Colombeau algebra $\mathcal{G}(\mathbb{R}^2)$ (see the Appendix)

$$\mathbf{u} = (u_\varepsilon)_\varepsilon + \mathcal{N}(\mathbb{R}^2) \tag{2}$$

and consider it as the generalized solution of (CIVP).

Some association processes of generalized functions with distributions are defined in $\mathcal{G}(\mathbb{R}^2)$. For example, for $u = [u_\varepsilon] \in \mathcal{G}(\mathbb{R}^2)$ (class of $(u_\varepsilon)_\varepsilon$), $T \in \mathcal{D}'(\mathbb{R}^2)$, Φ a mapping from $\mathbb{R}_+$ to $\mathbb{R}_+$ such that the class of $(\Phi_\varepsilon)_\varepsilon$ is a generalized number we define

$$u \sim T \Leftrightarrow \lim_{\substack{\varepsilon\to 0 \\ \mathcal{D}'(\mathbb{R}^2)}} u_\varepsilon = T$$

$$u \underset{\Phi}{\sim} T \Leftrightarrow \lim_{\substack{\varepsilon\to 0 \\ \mathcal{D}'(\mathbb{R}^2)}} \Phi(\varepsilon) u_\varepsilon = T.$$

Let us suppose $F = 0$ and $f \in \mathrm{C}^\infty(\mathbb{R}) \cap \mathrm{L}^1(\mathbb{R})$ with $\int f(x)dx = 1$. We know that if $f_\varepsilon(x) = \frac{1}{\varepsilon} f(\frac{x}{\varepsilon})$, then $\lim\limits_{\substack{\varepsilon \to 0 \\ \mathcal{D}'(\mathbb{R})}} f_\varepsilon = \delta$. Putting

$$\frac{1}{\varepsilon} u_\varepsilon(t,x) = \frac{1}{\varepsilon} f(\frac{x - \varphi(t)}{\varepsilon})$$

one can show that

$$u \underset{\varepsilon}{\sim} \delta_{\Gamma_\varphi}.$$

In other words, u have a bidimensional soliton structure, and $\operatorname{supp} u = \operatorname{supp} \delta_{\Gamma_\varphi} = \Gamma_\varphi$: the solution of the characteristic Cauchy problem for the unidirectional wave equation is associated to a bidimensional soliton whose support is the characteristic curve. In the general case ($F \neq 0$) the solution is associated to the sum of such a soliton and another generalized function corresponding to the nonlinearity.

Appendix : the sheaf of Colombeau simplified algebras

Let C^∞ be the sheaf of complex valued smooth functions on $\mathbb{R}^d$ ($d \in \mathbb{N}$), with the usual topology of uniform convergence. For every open set Ω of $\mathbb{R}^d$, this topology can be described by the family of semi norms

$$p_{K,l}(f) = \sup_{|\alpha| \leq l, K \Subset \Omega} |\partial^\alpha f(x)|,$$

where the notation $K \Subset \Omega$ means that the set K is a compact set included in Ω.

Let us set

$$\mathcal{E}_M(\Omega) = \left\{ (f_\varepsilon)_\varepsilon \in \mathrm{C}^\infty(\Omega)^{(0,1]} \mid \forall l \in \mathbb{N},\ \forall K \Subset \Omega,\ \exists q \in \mathbb{N},\ p_{K,l}(f_\varepsilon) = o(\varepsilon^{-q}) \text{ for } \varepsilon \to 0 \right\},$$

$$\mathcal{N}(\Omega) = \left\{ (f_\varepsilon)_\varepsilon \in \mathrm{C}^\infty(\Omega)^{(0,1]} \mid \forall l \in \mathbb{N},\ \forall K \Subset \Omega,\ \forall p \in \mathbb{N},\ p_{K,l}(f_\varepsilon) = o(\varepsilon^{p}) \text{ for } \varepsilon \to 0 \right\}.$$

We have the following

Lemma

i. The functor $\mathcal{F} : \Omega \to \mathcal{E}_M(\Omega)$ defines a sheaf of subalgebras of the sheaf $(\mathrm{C}^\infty)^{(0,1]}$.

ii. The functor $\mathcal{N} : \Omega \to \mathcal{N}(\Omega)$ defines a sheaf of ideals of the sheaf $\mathcal{F}$.

We shall not prove in detail this lemma but quote the two mains arguments:

(a) For each open subset Ω of X, the family of seminorms $(p_{K,l})$ related to Ω is compatible with the algebraic structure of $\mathrm{C}^\infty(\Omega)$; In particular:

$$\forall l \in \mathbb{N},\ \forall K \Subset \Omega,\ \ \exists C \in \mathbb{R}_+^*,\ \ \forall (f,g) \in (\mathrm{C}^\infty(\Omega))^2\ \ p_{K,l}(fg) \leq C p_{K,l}(f)\, p_{K,l}(g),$$

(b) For two open subsets $\Omega_1 \subset \Omega_2$ of $\mathbb{R}^d$, the family of seminorms $(p_{K,l})$ related to Ω_1 is included in the family of seminorms related to Ω_2 and

$$\forall l \in \mathbb{N},\ \forall K \Subset \Omega_1,\ \ \forall f \in \mathrm{C}^\infty(\Omega_2),\ \ p_{K,l}(f_{|\Omega_1}) = p_{K,l}(f).$$

This leads to the following

Definition

The sheaf of factor algebras

$$\mathcal{G}(\cdot) = \mathcal{E}_M(\cdot) / \mathcal{N}(\cdot)$$

is called the sheaf of *Colombeau simplified algebras*.

The sheaf $\mathcal{G}$ turns to be a sheaf of differential algebras and a sheaf of modules on the factor ring $\overline{\mathbb{C}} = \mathcal{F}(\mathbb{K}) / \mathcal{N}(\mathbb{K})$ with

$$\mathcal{F}(\mathbb{K}) = \left\{ (r_\varepsilon)_\varepsilon \in \mathbb{K}^{(0,1]} \mid \exists q \in \mathbb{N},\ |r_\varepsilon| = o(\varepsilon^{-q}) \text{ for } \varepsilon \to 0 \right\},$$

$$\mathcal{N}(\mathbb{K}) = \left\{ (r_\varepsilon)_\varepsilon \in \mathbb{K}^{(0,1]} \mid \forall p \in \mathbb{N},\ |r_\varepsilon| = o(\varepsilon^{p}) \text{ for } \varepsilon \to 0 \right\},$$

with $\mathbb{K} = \mathbb{C}$ or $\mathbb{R}$.

The sentinel method for mixed boundary condition dissipative problem with incomplete data

A. OMRANE

Laboratoire d'Analyse, Optimisation et Contrôle, AOC, Université des Antilles et de la Guyane, Campus Fouillole, 97159 Pointe-à-Pitre, Guadeloupe, France – aomrane@univ-ag.fr

(A part of this work can be found in [4])

***Abstract*— Many environmental problems contain incomplete data. We propose a sentinel method for the detection of pollution present in the state equation of a dissipative system of incomplete initial condition. In the present case, the control and the observation have their supports in different open sets.**
The problem of determining a sentinel is equivalent to a controllability problem, for which we use Carleman inequalities. We then prove the existence of a non nul sentinel.

Key words. Dissipative systems, sentinels, incomplete data, mixed boundary conditions.

I. Introduction

Let be $T > 0$, and Ω an open subset of $\mathbf{R}^d$ of regular boundary $\partial\Omega$, and denote by $Q = \Omega\times]0, T[$ the space-time cylinder. We are intersted in systems not completely known, where some of the conditions are not entirely available; we consider here the parabolic equation :

$$\left\{\begin{array}{rcll} y' - \Delta y + f(y) & = & \xi + \lambda\hat{\xi} & \text{in} \quad Q, \\ y(0) & = & y^0 + \tau\hat{y}^0 & \text{in} \quad \Omega, \\ y & = & 0 & \text{on} \quad \Sigma_1, \\ \dfrac{\partial y}{\partial \nu} & = & 0 & \text{on} \quad \Sigma_2, \end{array}\right. \tag{1}$$

where $y = y(x, t; \lambda, \tau)$, and where Σ_1 is a part of the boundary $\Sigma = \partial\Omega\times]0, T[$ and $\Sigma_2 = \Sigma\backslash\Sigma_1$. In the present situation we assume that $f \; : \mathbf{R} \rightarrow \mathbf{R}$ is a given application of class C^1, the functions ξ and y^0 are known with $\xi \in L^2(Q)$ and $y^0 \in L^2(\Omega)$. However, the terms : $\lambda\hat{\xi}$ (called pollution term) and $\tau\hat{y}^0$ (called perturbation term) are unknown. We only assume that

$$\|\hat{\xi}\|_{L^2(Q)} \leq 1, \quad \|\hat{y}^0\|_{L^2(\Omega)} \leq 1$$

where the reals λ, τ are small inough.

Let be now a non empty open subset $O \subset \Omega$, called observatory, and an observation of y on O, during the time T. We denote by y_{obs} this observation

$$y_{obs} = m_o \in L^2(O \times (0, T)). \tag{2}$$

We suppose that (1) admits a unique solution denoted

$$y(\lambda, \tau) := y(x, t; \lambda, \tau)$$

in a suitable space. The question is

> how to calculate the pollution $\lambda\hat{\xi}$ present in the state equation, independent of the variation $\tau\hat{y}^0$ around the initial data ? (3)

a) Least squares: Question (3) is natural since the main interest is the detection of the pollution term. One can use the least squares method. Here, we consider the unknowns $\{\lambda\hat{\xi}, \tau\hat{y}^0\} = \{v, w\}$ as control variables, then the state $y(x, t; v, w)$ has to be as close as possible to m_o.
Technically, this is an optimal control problem. With this method we search for v and w together, so there is no real possibility to separate the two variables.

b) Sentinels: The sentinels method of Lions [3] is a least squares method particularly adapted to the detection of pollution in ecosystems of incomplete data; many models can be found in litterature. It relies on the following three considerations : a state equation (for instance (1)), an observation function (2), and a control function w to determine.
This method is not the only one; we can distinguish the data assimilation method (a least squares method using inertial manifolds) and some other more or

less known methods.

Below, we consider the sentinel method of Lions which is an other attempt and brings better answer to question (3), as we will explain now :
Let h_0 be a given function in $L^2(O\times(0,T))$. Let besides ω be an open and non empty subset of Ω. For a control function w $\in L^2(\omega\times(0,T))$, we introduce the functional

$$S(\lambda,\tau)=\int_0^T\int_O h_0\,y(\lambda,\tau)\,dxdt + \int_0^T\int_\omega \mathrm{w}\,y(\lambda,\tau)\,dxdt. \quad (4)$$

We shall say that S defines a sentinel for the problem (1) if there exists w such that S is insensitive (at first order) with respect the to missing terms $\tau\hat{y}^0$ which means

$$\frac{\partial S}{\partial\tau}(0,0)=0 \quad (5)$$

for any $\hat{y}^0$ where here $(0,0)$ corresponds to $\lambda=\tau=0$, and if w has the property of minimal norm in the sense

$$\|\mathrm{w}\|_{L^2(\omega\times(0,T))} \quad = \quad \text{minimum}. \quad (6)$$

Remark *1:* The Lions sentinels S correspond to the case where $\omega=O$. In this case, the observation and the control have the same support, and so w $=-h_0$ is an evident solution of (5), and we only have to solve (6). We then have to be sure that the minimal solution of (6) is different from $-h_0$.
Many papers use the definition of Lions in the theoretical aspect as well as in the numerical one (see Kernevez [2]). Here we consider the case $\omega\neq O$. We then avoid the case w $=-h_0$.

c) Informations given by the sentinel: Because of (5) we can write

$$S(\lambda,\tau)\simeq S(0,0)+\lambda\frac{\partial S}{\partial\tau}(0,0), \qquad \text{for} \quad \lambda,\tau \quad \text{small}.$$

In (4), $S(\lambda,\tau)$ is observed and using (2), it equals

$$\int_Q(h_0\chi_O+\mathrm{w}\chi_\omega)m_o\,dxdt$$

so that (5) becomes

$$\lambda\frac{\partial S}{\partial\lambda}(0,0)\simeq\int_Q(h_0\chi_O+\mathrm{w}\chi_\omega)(m_o-y_0)\,dxdt, \quad (7)$$

with

$$\frac{\partial S}{\partial\lambda}(0,0)=\int_Q(h_0\chi_O+\mathrm{w}\chi_\omega)y_\lambda\,dxdt=0.$$

The derivative y_λ only depends on ξ and other known data. Consequently, the estimates (7) contains the informations on $\lambda\xi$.

II. Equivalence to a controllability problem

We shall show in this section that the existence of a sentinel is equivalent to a null controllability problem. We begin by transforming the insensibility condition (5).
Let us introduce

$$y_\tau=\frac{d}{d\tau}y(\lambda,\tau)\Big|_{\lambda=\tau=0}.$$

Then the function y_τ is given by

$$\left\{\begin{array}{lclll} y'_\tau-\Delta y_\tau+f'(y_0)y_\tau &=& 0 & \text{in} & Q,\\ y_\tau(0) &=& \hat{y}^0 & \text{in} & \Omega,\\ y_\tau &=& 0 & \text{on} & \Sigma_1,\\ \dfrac{\partial y_\tau}{\partial\nu} &=& 0 & \text{on} & \Sigma_2, \end{array}\right. \quad (8)$$

where $y_0=y(0,0)$. Problem (8) admits a unique solution y_τ under general assumptions on f: for example $f(y)=y^3$.

The insensibility condition (5) is equivalent to

$$\int_Q\left(h_0\chi_O+\mathrm{w}\chi_\omega\right)y_\tau\,dxdt \quad = \quad 0 \quad (9)$$

where χ_O and χ_ω denote the characteristic functions of O and ω respectively.
Denote by

$$L=\frac{\partial}{\partial t}-\Delta+f'(y_0)\,I_d \quad (10)$$

and by

$$L^*=-\frac{\partial}{\partial t}-\Delta+f'(y_0)\,I_d \quad (11)$$

its adjoint. We can transform (9) by introducing the classical adjoint state. More precisely, we define the function $q=q(x,t)$ as the solution of the backward problem :

$$\left\{\begin{array}{lclll} L^*q &=& h_0\chi_O+\mathrm{w}\chi_\omega & \text{in} & Q,\\ q(T) &=& 0 & \text{in} & \Omega,\\ q &=& 0 & \text{on} & \Sigma_1,\\ \dfrac{\partial q}{\partial\nu} &=& 0 & \text{on} & \Sigma_2. \end{array}\right. \quad (12)$$

As for the problem (8), the problem (12) also admits a unique solution q (under very general assumptions on $f'(y_0)$). The function q depends on w that we shall determine :

If we multiply the first equation in (12) by y_τ, and we integrate by parts over Q, we obtain

$$\int_Q \left(h_0\chi_O + \mathrm{w}\chi_\omega\right) y_\tau \, dxdt \;=\; \int_\Omega q(0)\tilde{y}^0 dx$$

for every $\tilde{y}^0$ such that $\|\tilde{y}^0\|_{L^2(\Omega)} \leq 1$. Hence, condition (5) (or (9)) is equivalent to

$$q(0) \quad = \quad 0. \tag{13}$$

Finally, (12) with (13) is a null-controllability problem.

III. Existence of a sentinel

We begin by giving an observability inequality. Consider

$$\mathcal{V} = \Big\{ v \in \mathcal{C}^\infty(\overline{Q}) \text{ such that } v_{|\Sigma_1} = \frac{\partial v}{\partial t}\Big|_{\Sigma_1} = \frac{\partial v}{\partial \nu}\Big|_{\Sigma_2} = 0 \Big\}. \tag{14}$$

Then we have :

Corollary *1:* Let be $u \in \mathcal{V}$ defined by (14), then there exists a positive constant $C = C(\Omega,\omega,O,T,f'(y_o))$ such that

$$\int_Q \frac{1}{\theta^2}|u|^2 dxdt \leq C\Big(\int_Q |L\, u|^2 dxdt + \int_0^T\int_\omega |u|^2 dxdt\Big) \tag{15}$$

where $\theta \in \mathcal{C}^2(Q)$ positive with $\dfrac{1}{\theta}$ a bounded weight function.

Proof - For the proof of this result which is based on the classical Carleman estimates for the heat equation, we refer to Imanuvilov [1] for the case $\Sigma_2 = \emptyset$ and $\Sigma_1 = \Sigma$, and to Miloudi *et* al. [4] in the more general case (14). ■

According to the RHS of (15), we consider the space $\mathcal{V}$ endowed with the bilinear form $a(.,.)$ defined by :

$$a(u,v) = \int_Q Lu\, Lv\, dxdt + \int_0^T\int_\omega u\, v\, dxdt. \tag{16}$$

Let V be the completion of $\mathcal{V}$ with respect to the norm

$$v \mapsto \|v\|_V = \sqrt{a(v,v)}, \tag{17}$$

then, V is a Hilbert space for the scalar product $a(v,\hat{v})$ and the associated norm. Moreover, $\mathcal{V}$ is dense in V.

Remark *2:* We can precise the structure of the elements of V. Indeed, let $H_\theta(Q)$ be the weigthed Hilbert space defined by

$$H_\theta(Q) = \Big\{ v \in \mathcal{V} \text{ such that } \int_Q \frac{1}{\theta^2}|v|^2 dxdt < \infty \Big\}$$

endowed with the natural norm $\|v\|_\theta = (\int_Q \frac{1}{\theta^2}|v|^2 dxdt)^{\frac{1}{2}}$. Then the left hand side in (15) shows that V is imbedded continuously in $H_\theta(Q)$. In the same way, we have

$$\|v\|_\theta \leq C\,\|v\|_V. \tag{18}$$

Now if $h_0 \in L^2(Q)$ and $\theta h_0 \in L^2(Q)$ (i.e : $h_0 \in L^2_\theta(Q)$), then thanks to (15) and the Cauchy-Schwartz inequality, we deduce that the linear form defined on V by

$$v \mapsto \int_Q h_0\chi_O\, v\, dxdt$$

is continuous.

Therefore, from the Lax-Milgram theorem there exits a unique u in V solution of the variational equation :

$$a(u,v) = \int_Q h_0\chi_O\, v\, dxdt \qquad \forall v \in V. \tag{19}$$

Theorem *1:* Let be $h_0 \in L^2_\theta(Q)$, and let u be the unique solution of (19). We set

$$\mathrm{w} = -u\chi_\omega \qquad \text{and} \qquad q = Lu.$$

Then, the pair (w,q) is such that (12)-(13) hold (i.e. then there exists an insensitive sentinel defined by (4)-(5)).

■

References

[1] O. Yu. Imanuvilov Controllability of parabolic equations, Sbornik Mathematics 186 : 6, 1995, pp 879 − 900.

[2] Kernevez J.-P., The Sentinel Method and its Application to Environmental Pollution Problems, CRC Press, Boca Raton, FL, 1997.

[3] Lions J.-L. Sentinelles pour les systèmes distribués à données incomplètes, Masson, Paris, 1992.

[4] Miloudi Y., Nakoulima O., Omrane A., A method of detecting pollution in dissipative systems of incomplete data, ESAIM:Proceeding, Vol 17, 2007, pp 67 − 79.

[5] Nakoulima O., Contrôlabilité à zéro avec contraintes sur le contrôle. C. R. Acad. Sci. Paris, Ser. I 339, 2004, pp 405 − 410.

Angles & Logarithms

ALAIN B. TORRENS

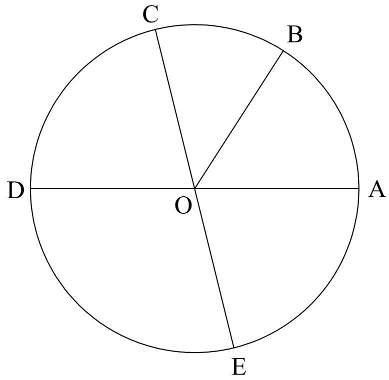
Université Quisqueya, BP 796, Port-au-Prince, Haïti - torrab@yahoo.com

Introduction

Angles are physical quantities endowed with a dimension [0], as indicated by the fact that they are expressed in some unit - typically, the degree (°), radian (rad), or revolution (r). For example, one will quote an angle of '36°', or '0.628 rad', but not '0.628'.

On the other hand, a truly dimensionless quantity is expressed as a simple number: an efficiency could be written as '0.62', or '62 %'; a gain (ratio of two voltages) could be equal to 15.8, and an index of refraction, 1.42.

Fig·0



Two angles can be compared, added or subtracted; e.g. , in Fig·0:

B**O**C<C**O**D = A**O**E; A**O**C = A**O**B + B**O**C;

A**O**E = B**O**E − B**O**A;

But one cannot compare an angle with a number, which should be possible if, as some assert [1, Table 3, Note b; sec.2.2.3], angles have no dimension. Furthermore, if angles were dimensionless, their unit would be the unit number 1 (one).

The angle as a fundamental quantity

Not only can angles not be compared with numbers; they cannot be compared with, or added to, any other physical quantity, such as force, time, volume, etc… This implies that they are a **fundamental quantity**, on a par with mass, length, time,…

Particular angles

The **radian** is defined as the angle subtended at the center of a circle by an arc of circumference equal in length to the radius of the circle. (Notice that the radian is defined as an *angle*, not as the ratio of two lengths.)

The natural unit of angle is the **revolution** r. It is the minimum, non-zero, angle of rotation about O that transforms a point M into itself.

The **straight angle** is equal to the half a revolution: $\Pi = \frac{1}{2}\,\mathrm{r} = 180° = \pi{\cdot}\mathrm{rad}$ **:1**.

Notice the distinction between the *angle* Π and the *number* π. With this notation, the revolution may be noted '2Π' (not 2π), the right angle 'Π/2', etc.

The angular constant

Let us consider the angle $\theta = \mathrm{A}\mathbf{O}\mathrm{B}$ in Fig·0; the length s of the arc AB subtended by the angle θ is, traditionally, written as

$$s = R\theta \qquad \textbf{:2}.$$

Since the angle θ has a dimension A (just as the radius R has the dimension L of a length), this equation is dimensionally incorrect, since it implies that the dimension of s is $[s] = \mathsf{LA}$ instead of L.

In fact, s is not *equal* to $R\theta$, but only *proportional* to it, through a factor η called **angular constant** [0]:

$$s = \eta R\theta \qquad \textbf{:3}.$$

The value of η is determined by the lengths of particular arcs:

- If θ = r, the arc is a complete circle: $2\pi\mathrm{R} = \eta R{\cdot}\mathrm{r} \;\Rightarrow\; \eta = 2\pi/\mathrm{r}$;
- If θ = rad, by definition of the radian, $s = R = \eta R{\cdot}\mathrm{rad} \;\Rightarrow\; \eta = \mathrm{rad}^{-1}$.

With Rel·1, we summarize:

$$\eta = \frac{2\pi}{r} = \frac{\pi}{\Pi} = \frac{1}{rad} \approx 0.01745\ (°)^{-1} \qquad \textbf{:4}.$$

The question arises: If Rel·2 is incorrect, how has it been used so often, and for so long? The answer: with many cautions and arbitrary rules, the basic one being: *all angles must be in radians*. Why this rule? Because Rel·2 should be written

$$s = Ru \qquad \text{:5},$$

where u is not the *angle* θ but its *measure in radian*. Indeed, in Rel·3, one can replace η by its expression with respect to the radian:

$$\eta\theta = \theta/\mathrm{rad}.$$

Unlike Rel·2, Rel·5 is correct (since the measure u of θ in radian is, like all measures, dimensionless), but it has the drawback of being a *mixed equation*: both a *quantity equation* (independent of any unit that may eventually be used to express s and R) and a *measure equation*, since it is valid only with the radian as unit of angle. On the other hand, Rel·3 is a true quantity equation, independent of *any* unit.

One can also define the **analytical angle** [0] u not as an the measure of θ in radian, but as the product $\eta\theta$:

$$u == \eta\theta \qquad \text{:6}.$$

Rel·3&6 ⇒ Rel·5. In this perspective, u is a new variable which *happens* to be equal to the measure of θ in radian but is really independent from it, since η is (like Planck's constant or the celerity of light in vacuum) a *physical constant* which preexists any unit, and which can be expressed in units other than the radian - Rel·4.

If u is defined by Rel·6, then Rel·3 et Rel·5 are equivalent. However, it must be emphasized that u is *not* an angle (in the usual sense of the word, adopted in this paper) but a pure number, and should be expressed as such, without any unit (other than the number 1 [1, Sec.2.2.3]) —not even the radian, which is unit of *angle*, not of number.

Confusing the geometrical angle θ and the analytical angle u leads to confusion of their respective units and bizarre conclusions. For example, according to the BIPM, the radian, classed as a 'supplementary unit' from 1960 to 1995, is now a derived unit equal to m/m (m = mètre). Since m/m = 1, then rad = 1, and 'radian' is 'special name for the number one' [1, Sec.2.2.2; Table 3, note b]. It this is true, then one can write a length of two metres as

$$a = 2\ \mathrm{m} = 2\ \mathrm{m{\cdot}rad} = 2\ \mathrm{m/rad^2} = \ldots!.$$

In fact, the radian is a *fundamental* (not derived) unit of *angle*, distinct from the *number* 1, which is a most fundamental unit, not a 'dimensionless derived unit' [1, Sec.2.2.3; Table 3, note b], since numbers exist independently of any physical quantity. (Is the number 1 'derived' from the meter, by m/m = 1, from the second, by s/s = 1, or from the coulomb, by C/C = 1?)

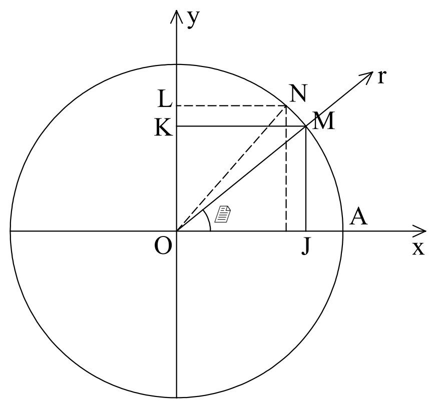
Trigonometric functions

To define the cosine and sine of an angle $\theta = (\mathbf{Ox},\mathbf{Or})$, one considers the intersection M of Or with the trigonometric circle, and the coordinates of M (Fig·1):

$$\mathrm{Cos}\theta = x = \underline{OJ} \qquad \text{:7},$$

$$\mathrm{Sin}\theta = y = \underline{OK} \qquad \text{:8}.$$

Fig·1



To evaluate the derivative of Sinθ, let us consider a variation $\Delta\theta$ of θ. Point M moves to N, and its projection K moves to L. Since the radius of the trigonometric circle is 1, Rel·3 ⇒

$$s = AM \overset{3}{=} \eta\theta, \quad \text{and} \quad MN = \Delta s = \eta\Delta\theta,$$

so that $\Delta y = \underline{KL} = MN{\cdot}\mathrm{Cos}\,(\mathbf{Oy},\mathbf{MN}) \qquad \text{:9},$

where

$(\mathbf{Oy},\mathbf{MN}) = (\mathbf{Oy},\mathbf{Ox}) + (\mathbf{Ox},\mathbf{OM}) + (\mathbf{OM},\mathbf{MN})$

$= -\Pi/2 + \theta + \Pi/2 = \theta$

Rel·9&10 ⇒ $\Delta y/\Delta\theta = \eta{\cdot}\mathrm{Cos}\theta \qquad \text{:11}.$

The limit value of this ratio when $\Delta\theta \rightarrow 0$ is the **derivative** of Sinθ: $\dfrac{d\,Sin\theta}{d\theta} = \eta\cdot\mathrm{Cos}\theta$ **:12.**

The above proof was developed in detail to show how the angular constant η appears in Rel·12. According to this relation, the dimension of d(Sinθ)/dθ is that of η, namely A^{-1}. This is consistent with the definition, since the ratio $\Delta y/\Delta\theta$ (Rel·11) is a number divided by an angle.

To reconcile Rel·12 with the well-known formula $\dfrac{d\,Sin\,u}{du} = \cos u$ **:13**,

one must realize that the mathematical functions sin (u), cos (u), tan (u) require a (dimensionless) *analytical angle* as argument, while the angular functions Sinθ, Cosθ, Tanθ require an *angle* as argument. In other words, their respective **domains** [2] are the set of **reals** (**R**) and the set of **angles**; they share the same **codomain**, since their values are reals. They satisfy the identities

$$\mathrm{Sin}\,(\theta) \equiv \sin\,(\eta\theta);$$
$$\mathrm{Cos}\,(\theta) \equiv \cos\,(\eta\theta);$$
$$\mathrm{Tan}\,(\theta) \equiv \tan\,(\eta\theta) \quad \textbf{:14}.$$

Common practice is to confuse these functions and write, for example, sin (30°) = ½, which is incorrect since the angle 30° is not an acceptable argument for the function sin; instead, one could write: sin (π/6) = Sin (π/6 rad) = Sin (30°) = ½.

With Rel·6, Rel·13 & 14a

$$\Rightarrow \frac{d\,Sin\theta}{d\theta} = \frac{d\,Sin\,u}{du} \times \frac{du}{d\theta}$$

$$=^{6} \eta\cdot\cos\,(u) = \eta\cdot\mathrm{Cos}\,(\theta),$$

which confirms Rel·12. Similarly,

$$\frac{d\,Cos\theta}{d\theta} = \eta\cdot\frac{d\,Cos\,u}{du} = -\eta\cdot\mathrm{Sin}\theta \quad \textbf{:15},$$

and $\dfrac{d\,Tan\,\theta}{d\theta} = \eta\cdot\dfrac{d\,Tan\,u}{du} = \eta/\mathrm{Cos}^2\theta.$

Conversely, the **primitive** of Cosθ is

$$\int \mathrm{Cos}\theta\cdot d\theta = \eta^{-1}\cdot\mathrm{Sin}\theta \quad \textbf{:16};$$

its dimension is that of dθ (left-hand side of Rel·16) and η^{-1} (right-hand side), namely A.

Frequency; angular frequency

Let us consider a point M in uniform motion on a circle of radius R (Fig·1): $\theta = \Omega t$ **:17**.

The **period** is the duration of one revolution:

$$T = \mathrm{r}/\Omega \quad \textbf{:18},$$

the **frequency** of the movement is

$$f = T^{-1} = \Omega/\mathrm{r} \quad \textbf{:19}.$$

and the **angular velocity** is

$$d\theta/dt = \Omega = \mathrm{r}\cdot f = 2\Pi\cdot f \quad \textbf{:20}.$$

The projection J of M on the x-axis is at

$$x = R\cdot\mathrm{Cos}\theta = R\cdot\mathrm{Cos}\Omega t \quad \textbf{:21},$$

and its velocity is, with Rel·15,

$$v_x = dx/dt = -R\cdot\eta\Omega\cdot\mathrm{Sin}\Omega t = -v_{max}\cdot\mathrm{Sin}\Omega t \quad \textbf{:22},$$

where, with Rel·4b & 19,

$$v_{max} == RW \quad \textbf{:23},$$

where $W == \eta\Omega = 2\pi f$ **:24**.

Example: If f = 30 Hz; Rel·19

$\Rightarrow \Omega$ = 30·rHz = 30·r/s = 1800 r/min **:25**.

If R = 40 cm, Rel·23,24,25

$\Rightarrow v_{max}$ = 40 cm×2π×30 Hz = 75.4 m/s

Traditionally, one would set the factor η to unity (as in Rel·2), confuse W with Ω (Rel·24), write Ω = 2π·f ≈ 188.5 Hz = 188.5 s−1,

add 'we know that an angular velocity must be in rad/s', then conclude: Ω = 188.5 rad/s.

Then, one would calculate

$v_{max} = R\cdot\Omega$ = 40 cm×188.5 rad/s = 75.4 m·rad/s,

add 'we know that a velocity must be in m/s' and 'drop' (arbitrarily) 'rad' from the result.

Not only are such questionable acrobatics (introducing or removing the radian where that seems appropriate) not required to apply Rel·19 & 23, but such a result as Rel·25 involves *practical* units — r/s, r/min. (Even to the mathematically-inclined, a speed of 188.5 rad/s is difficult to grasp.)

Circlance

The voltage $V = V_{max}{\cdot}\text{Sin}\,\omega t$ **:26**

varies with time t with a frequency f and an **angular frequency** $\omega = 2\Pi f = \text{r}\,f$ **:27**.

This voltage, applied to a capacitor of capacitance C, produces an electric current of intensity

$$i = C{\cdot}\text{d}V/\text{d}t;$$

with Rel·12,
$i = C{\cdot}V_{max}{\cdot}\eta\omega{\cdot}\text{Cos}\,\omega t = Y{\cdot}V_{max}{\cdot}\text{Cos}\,\omega t$,

where Y is the admittance: $Y == wC$ **:28**,

Where $w == \eta\omega = 2\pi f$ **:29**.

Example: If $f = 10$ kHz and $C = 47$ nF, Rel·29

$\Rightarrow w = 62.8$ kHz, and Rel·28$\Rightarrow$ $Y = 2.95$ mS.

Traditionally, one would write:

$\omega = 2\pi f = 62.8\times10^3\ \text{s}^{-1}$,

and express the result as 62.8×10^3 rad/s (because 'ω must be in rad/s'),

then calculate $Y == \omega C$

$= (62.8\times10^3\ \text{rad/s})\times(47\times10^{-9}\text{F}) = 2.95$ rad·mS,

then drop 'rad' from the result because it obviously does not belong there…

W (Rel·24) or w (Rel·29) are similar quantities that have the dimension T^{-1} of a frequency, but are not *the* frequency f; they are usually called 'angular velocity' or 'angular frequency', but these terms apply to Ω and ω (Rel·20,27), which have the angular dimension AT^{-1}. The quantity $2\pi f$ associated with a sinusoidal oscillation of frequency f could be called **circlance** (from the verb 'to circle'), by analogy with the circumference of a circle of radius R: c = 2πR.

Remark

Different quantities have different names and symbols, yet may have the same dimension and, *consequently*, the same unit. For example, Rel·29 implies that the frequency f and the circlance w have the same dimension T^{-1}, and therefore the same unit Hz. One should *not* use a special unit for w just to show that it is different from f, as suggested by some, including the BIPM [1, Sec.2.2.2]. (In particular, the unit rad/s is applicable to ω, but not to w.) Similarly, the radius R, diameter D, and circumference c of a circle all have the same dimension (L) and unit (m, cm, ft,…).

Angular exponential

Consider an angle θ and another real quantity ξ of the same dimension A. The complex quantity

$$\zeta == \xi + \text{i}{\cdot}\theta \qquad \textbf{:30}$$

also has the dimension A, because the imaginary unit i is a pure number. (The quantities ζ, ξ and θ must, by homogeneity, have the same dimension for the expression $|\zeta|^2 = \xi^2 + \theta^2$ to be meaningful.)

On ther other hand, $x == \eta\xi$ **:31,**

$$u == \eta\theta \qquad \textbf{:32}$$

and $g == \eta\zeta = x + \text{i}{\cdot}u$ **:33**,

are dimensionless numbers; as such, they are acceptable as arguments of the exponential function. Consistently with Rel·14, the **angular exponential** function Exp (capitalized, to distinguish it from the numeric function exp) is defined over the angle domain by

$\text{Exp}\,(\zeta) == \exp\,(g) = \exp\,(x + \text{i}u)$

$= \exp\,(x){\cdot}\exp\,(\text{i}u) = \exp\,(x){\cdot}(\cos u + \text{i}{\cdot}\sin u)$

In particular, for $\zeta = \xi$, $\text{Exp}\,(\xi) = \exp\,(x)$ **:36**

and, for $\zeta = \text{i}{\cdot}\theta$,

$\text{Exp}\,(\text{i}{\cdot}\theta) == \exp\,(\text{i}{\cdot}u)$

$= \cos u + \text{i}{\cdot}\sin u = \text{Cos}\,\theta + \text{i}{\cdot}\text{Sin}\,\theta$ **:37**.

Rel·35 may therefore be written

$\text{Exp}\,(\xi + \text{i}{\cdot}\theta) \equiv \text{Exp}\,(\xi){\cdot}\text{Exp}\,(\text{i}{\cdot}\theta)$

$\equiv \text{Exp}\,(\xi){\cdot}(\text{Cos}\,\theta + \text{i}{\cdot}\text{Sin}\,\theta)$ **:38**.

Note: in Rel·35, exp (g) may be calculated as e^g (where the pure number g is an acceptable power), but e^ζ (where ζ has a dimension) is not defined, and may not be used as a substitute for Exp (ζ).

Angular logarithm

The complex natural logarithm is defined as the inverse function of the complex *numeric* exponential:

$$z = \exp(g) = \exp(x + \mathrm{i}u)$$

$$\Leftrightarrow g = \ln z = x + \mathrm{i}\cdot(u + n\cdot 2\pi) \qquad \textbf{:39},$$

where n is any integer.

The inverse function, noted Log, of the angular exponential is similarly defined: if z == Exp (ζ), Rel·33 ⇒ Log z = ζ = 33 η−1·g = 39 η−1·ln z

= η−1·[x + i·(u + n·2π)]

= 31,32 ξ + i·(θ + n·r) **:40**.

Just as ln z is defined modulo i·2π (where π is a *number*) (Rel·39), Log z is defined modulo i·r, where the revolution r is an *angle*:

Rel·1 ⇒ r = 2Π = 360°. From here on, we will consider the principal value of Log z, defined by $-\Pi < \theta \leq \Pi$ and $n = 0$ (Rel·40).

The **angular logarithm** Log is therefore a *quantity*, defined intrinsically, having the dimension A of η^{-1}, ζ, ξ and θ (Rel·40); it is also called 'indefinite logarithm' by Frank_MP, who stresses that 'no specific base is implied at all' in its definition [3]. It is 'angular' only in a broad sense: while the imaginary part (θ) of ζ is an angle; the real part ξ is of a different nature - see below -, even though it has the same dimension.

Since η^{-1} = rad (Rel·4), Rel·40b

$$\Rightarrow \mathrm{Log}\, z = \ln z \text{ rad} \qquad \textbf{:41}.$$

Therefore, the measure of Log z in radian is ln z. However, Rel·41 should be regarded as an *interpretation* (in terms of a particular unit, the radian) of Log z; the definition (Rel·40) is intrinsic, independent of any unit.

Conclusion

►Using the angular constant (Rel·4) instead of the number 1, and the angle Π (Rel·1) instead of the number π, where required (Rel·3, 12, 15, 16, 20, 27) removes all inconsistencies in relations between rotational and translational quantities, and obviates the need to introduce or delete the radian (or neper, decibel, etc.) 'where appropriate' [1, Sec.2.2.2, Table 3, Note b]. It also removes the requirement that angles be expressed in radians in some equations and permits, instead, the use of practical angle units such as the revolution or the degree, without the need for an explicit conversion into radians. In fact, it lets one write any relation intrinsically, in dependently of any unit.

►As an unit of angle, the **radian** is impractical. In real-life calculations, η is always expressed as 2π/r or π/180° (Rel·4); of course, expressing it as rad^{-1} simplifies the calculation (since the measure of η is, then, 1), but only temporarily, since a result such as 377 rad/s is so obscure that it must be converted (via another calculation) to a more comprehensible form, such as 60 r/s, or 3600 r/min.

In the end, the radian becomes useless because, as geometrical angles and the anguler constant are used systematically, there is no more need to convert angles into radians. In hindsight, we can see that the radian was invented just to avoid the use of the angular constant.

►The definitions of angular trigonometric and exponential functions lead to the conclusion that neper and radian are actually equal (Rel·45), with the consequence that one should be abandoned in favour of the other. I suggest keeping the neper, because:

- ☐ Naming units after scientists has been the recent trend;
- ☐ The name must be 'neutral' to apply to quantities as diverse as angles, logarithmic gains, entropy; 'radian' is perceived strictly as an angle unit, as suggested by its etymology [8].
- ☐ The neper will be used mostly for the real part of logarithm (as it currently is), seldom to express angles −see above.

►Since angles and logarithms constitute a fundamental quantity, of dimension A, the **neper** (so far, not even recognized as an SI unit [1, Sec.4.1, Table 8]) should become a **base unit** of the SI [1, Sec.2.1].

►While the angular frequency (Rel·27) has dimension AT−1 and is expressed in r/s (or r·Hz, or °/s, etc.), the circlance w (Rel·29) has dimension T−1 of a frequency and is expressed in Hz, not in rad/s. It is used (instead of ω) to express the susceptance of a capacitor or the reactance of a coil: B = wC, X = wL.

►The angular functions have the general properties of the traditional mathematical functions, so that one need not 're-learn' mathematics; for example:

Cos $(\alpha + \beta) \equiv \text{Cos}\alpha\cdot\text{Cos}\beta - \text{Sin}\alpha\cdot\text{Sin}\beta$;
Exp $(\alpha + \beta) \equiv \text{Exp}\alpha\cdot\text{Exp}\beta$;
Log $(ab) \equiv \text{Log}\ a + \text{Log}\ b$.

Sin $\theta \equiv \sin u = u - u^3/3! + u^5/5! - u^7/7! + \ldots$,

Where $u == \eta\theta$.

►It is possible (and permitted!) to prefer analytical (numerical) angles to geometrical ones. This simplifies the language (all 'angles' are, by convention, numerical) and mathematical formulas (by replacing Rel·3 with Rel·5, Rel·12 with Rel·13, etc.), and eliminates the need for such angular functions as Cos, Exp and Log.
By denying the existence of geometrical angles, this viewpoint even eliminates the need for, and use of, angular units - *even the radian*. It, therefore, forces one to quote an angle as a number (which it *is*), as '$\pi/3$' or '1.2' ─at the expense of clarity, because most people understand angles as geometrical entities. To make the expression clearer, one would therefore express the angle of '1.2' as '1.2 rad', thereby switching (implicitly) to the concept of geometrical angles.

References

0: Torrens A B: 'On angles and angular quantities', Metrologia 22, 1-7; 1986 Jan. See also Metrologia 23, 155; Metrologia 23, 57; Metrologia 24, 155.

1: BIPM (Bureau international des Poids et Mesures): 'The International System of Units (SI)', 8th edition (1998): www1.bipm.org/en/si/si_brochure/

2: http://en.wikipedia.org/wiki/Domain_%28 mathematics%29.

3: Frank_Michael_P., The Indefinite Logarithm, Logarithmic Units, and the Nature of Entropy, http://www.eng.fsu.edu/~mpf/IndefLog_p aper/indef-log.pdf#search='logarithm%20units'

4: http://en.wikipedia.org/wiki/Neper

5: http://en.wikipedia.org/wiki/Ban_%28info rmation%29

6: http://PlanetMath.org/encyclopedia/Shann onsTheoremEntropy.html

7: http://en.wikipedia.org/wiki/Entropy_%28 statistical_views%29.

8: http://members.aol.com/jeff570/r.html

Estimation of the Large Deviation Multifractal Spectrum

M. ABADI AND E. GRANDCHAMP

GRIMAAG UAG, Campus de Fouillole, BP 250, 97157 Pointe-à-Pitre Cedex,
Université Antilles Guyane, Guadeloupe, France - mabadi@univ-ag.fr - egrandch@univ-ag.fr

Keywords. multifractal theory, multifractal spectrum, quasi-continuous histogram, kernel, methods.

Abstract

Within the framework of the multifractal analysis, we propose to use the quasi-continuous histogram (QCH) method for the estimation the probability density in order to be able to compute the large deviation spectrum (LDS). The QCH is a statistical tool related to the kernel for density estimation. The application of this tool and the comparison with kernel methods on binomial measures is a new point in this field.

A Variant of Newton's Method for Generalized Equations

C. JEAN ALEXIS AND A. PIETRUS

Laboratoire AOC, Université des Antilles et de la Guyane, Département de Mathématiques et Informatique, Campus de Fouillole, F–97159 Pointe à Pitre, France
celia.jean-alexis@univ-ag.fr, apietrus@univ-ag.fr

Keywords: Set–valued mapping, generalized equation, linear convergence, Aubin continuity.

Abstract

Throughout this statement X and Y are two Banach spaces. We consider a generalized equation of the form

$$0 \in f(x) + F(x) \tag{1}$$

where $f : X \to Y$ is Frechet-differentiable and $f : X \to 2^Y$ is a set-valued map with closed graph. Let us note that the equation (1) is an abstract model for various problems.

- When $F = 0$, (1) is an equation,
- when F is the positive orthant in R^m, (1) is a system of inequalities,
- when F is the normal cone to a convex and closed set in X; (1) may represent variational inequalities.

Let us remark that when $F = \{0\}$ and x^* is a solution of (1) of order $h > 1$; the Newton method of the form

$$0 \in f(x_k) + \nabla f(x_k)(x_{k+1} - x_k) + F(x_{k+1}), \quad k = 0,1... \tag{2}$$

is no longer valid. To avoid this drawback, in [1, 2] the authors proposed a variant of the Newton method of the form

$$x_{k+1} = x_k - h\nabla f(x_k)^{-1} f(x_k) \tag{3}$$

Following this work, we introduce to solve (2), the following sequence of the form

$$0 \in f(x_k) + h\nabla f(x_k)(x_{k+1} - x_k) + F(x_{k+1}), \quad k = 0,1... \tag{4}$$

Let us remark that when $h = 1$, the method (4) is exactly the Newton type method (2). This statement is organized as follows : at first time, we recall a few preliminary results then in the second time, we show that the method (4) is locally convergent and to finish, we prove the stability of this method.

References:

[1] J.M. Ortega and W.C Rheinboldt, Iterative solution of nonlinear equations in several variables, Academic Press, New-York and London, 1970.

[2] A.M Ostrowski, Solution of equations in Euclidean and Banach spaces, Academic Press, New-York and London, 1970.

An Accurate Projection Method for Incompressible Flows

C. FEVRIERE† (1), P. ANGOT (2), J. LAMINIE (1), P. POULLET (1)

(1) *GRIMAAG, Groupe de Recherche en Informatique et en Mathématiques Appliquées des Antilles et de la Guyane, Université des Antilles et de la Guyane, Campus de Fouillole, 97159 Pointe-à-Pitre Cedex, Jacques.Laminie@univ-ag.fr , Pascal.Poullet@univ-ag.fr*
(2) *LATP, Laboratoire d'Analyse, Topologie et Probabilités, Université de Provence, 39 rue Joliot-Curie, 13453 Marseille Cedex 13, angot@cmi.univ-mrs.fr*

Abstract

The most important dif_culty for the numerical simulation of imcompressible _ows is that the velocity and the pressure are coupled by the imcompressibility constraint. Since the precursory works of Chorin and Temam [1, 8], several papers have treated about this subject [7, 6, 3], (and [2] for an overview), but research on this topic remains still active [4]. Our study is concerned with the implementation and study of projection methods, known to offer
a methodology to solve such a problem. The most attractive feature of projection methods is that, at each time step one only needs to solve a sequence of decoupled elliptic equations for the velocity and the pressure, making it very ef_cient for large scale numerical simulations. We focus our attention on the pressure correction schemes and particularly, the capabilities of the implementation with staggered mesh, to compute accurate solutions of problems with realistic boundary conditions. We study the incremental and penality projection methods in theirs standard and rotational forms. A comparative study of the four schemes for the time-dependent Stokes problem will help users to select the projection methods using MAC mesh which is more accurate.

Références

[1] A.J. Chorin, Numerical solution of the Navier-Stokes equations, *Math. Comput.* 22, pp. 745-762, (1968).
[2] J.L. Guermond, P. Minev and J. Shen, An overview of projection methods for incompressible flows *Comput. Methods Appl. Mech. Engrg.* (to appear).
[3] J.L. Guermond, L. Quartapelle, An the approximation of the unsteady Navier-Stokes equations by finite element projection methods *Numer. Math.* 80, pp. 207-238, (1998).
[4] M. Jobelin, C. Lapuerta, J.-C. LatchÈ, Ph. Angot and B. Piar A Finite Element penalty-projection method for incompressible flows, *J. Comput. Phys.* (to appear).
[5] P. Poullet, Staggered Incremental Unknowns for solving Stokes and generalized Stokes problems, *Applied Num. Math.* 35(1), pp. 23-41, (2000).
[6] J. Shen, On error estimates of projection methods for Navier-Stokes equations : Second-order schemes, *Math. Comput.* 65(215), pp. 1039-1065, (1996).
[7] J. Van Kan, A second-order accurate pressure-correction scheme for viscous incompressible flow, *SIAM J. on Scient. Stat. Comput.* 7(3), pp. 870-891, (1986).
[8] R. Temam, Sur l'approximation de la solution des équations de Navier-Stokes par la méthode à pas fractionnaires (II), *Archive for Rat. Mech. and Anal.* 33, pp. 377-385, (1969).

Introduction to and Implications of Emergent Global Internet Protocols and Policies for the Caribbean Region

J. Crain

Crain, John – Chief Technical Officer ICANN - Internet Corporation for Assigned Names and Numbers, Marina Del Rey, USA - john.crain@icann.org

The global internet continues to expand around the world. More than one billion people now have access to this network of networks. In addition to adding users, increasing demand for mobile internet access and convergence between Internet protocols, supply chain and other resource management systems is creating the potential for an internet of things.

This rapidly evolving landscape presents both opportunities and challenges for the science and technology sector. How are the systems that keep the Internet functioning responding to these changes? How are these systems governed? What are the potential implications for small island states and the scientific community?

This presentation will provide a user-friendly overview of basic Internet functions. It will also introduce some of the key systems and policies under discussion at a global level and outline how they are agreed on. Topics will include:

IPv6 and the extension of Internet addresses into physical space.
Internationalised domain names at the top level: developing a multi-lingual Internet infrastructure
Top level domain name current policy issues.
Bottom up policy making: who runs the Internet? You do.

Kernel Theorems in Spaces of Caribbean Generalized Functions

A. DELCROIX

Equipe AANL - Laboratoire AOC - Faculté des sciences exactes et naturelles
Université des Antilles et de la Guyane - 97110 Pointe-à-Pitre – GUADELOUPE & IUFM de Guadeloupe - Morne Ferret - BP 517 - 97178 Abymes Cedex – GUADELOUPE
Antoine.Delcroix@univ-ag.fr

Abstract

During the three last decades, theories of non-linear generalized functions have been developed by many authors, mainly based on the ideas of J.-F. Colombeau, which we are going to follow in the sequel. Those theories appear to be a natural continuation of the distributions' one, specially efficient to pose and solve differential or integral problems with irregular data. We continue here the investigations in the field of generalized integral operators initiated by D. Scarpalézos and carried out by various authors (S. Bernard, J.-F. Colombeau, A. Delcroix, C. Garetto, V. Valmorin). The importance of these operators is that they exactly generalize, in the Colombeau framework, the operators with distributional kernels in spaces of distributions.

More specifically, we show that any moderate net of linear maps - that is satisfying some growth properties with respect to a small parameter - gives rise to a linear map L : $\mathcal{G}_C(\mathrm{R}^n) \to \mathcal{G}(\mathrm{R}^m)$ ($\mathcal{G}_C(\mathrm{R}^d)$ and $\mathcal{G}(\mathrm{R}^d)$ denote respectively the space of generalized functions and the space of compactly supported ones.) The main result is that L can be represented as a generalized integral operator in the spirit of Schwartz Kernel Theorem. Moreover, through now classical results of embeddings of $\mathcal{D}(\mathrm{R}^n)$ (*resp*. $\mathcal{D}'(\mathrm{R}^m)$) into $\mathcal{G}_C(\mathrm{R}^n)$ (*resp*. $\mathcal{G}(\mathrm{R}^m)$), we show that the classical kernel theorem is contained in our result.

Going further in this direction, we show that the generalization of the classical isomorphism theorem between $\mathcal{S}'(\mathrm{R}^{n+m})$ and the space of continuous linear mappings acting between $\mathcal{S}(\mathrm{R}^n)$ and $\mathcal{S}'(\mathrm{R}^m)$ is possible. In this case, the spaces of generalized functions considered are the space $\mathcal{G}_S(\mathrm{R}^n)$ of rapidly decreasing generalized functions (in which $\mathcal{S}(\mathrm{R}^n)$ is embedded) and the space $\mathcal{G}_\tau(\mathrm{R}^m)$ of tempered generalized functions (in which $\mathcal{S}'(\mathrm{R}^m)$ is embedded). In this case, we also obtain a kernel result for moderate nets. Once more, the generalized kernel theorem contains the classical one.